NORTH-HOLLAND

A Neurofuzzy Approach to Variational Problems by Using Gaussian Membership Functions

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ABSTRACT

In this paper we propose a neurofuzzy direct solution method for variational problems in which the cost function of an integral form is minimized. We deal with two nonlinear systems; one is a direct drive (DD) manipulator system, and the other is a trailer-truck system. The DD manipulator system is described by a continuous-time dynamical model, and the trailer-truck system is described by a discrete-time dynamical model. The problem is to find trajectories which minimize the cost function of an integral form. The trajectories of state variables and input variables are represented by fuzzy models that consist of Gaussian membership functions. The networks of Gaussian functions are trained by the steepest-descent method to minimize the cost function. The proposed neurofuzzy approach provides a direct solution method of the variational problems by using Gaussian functions. The function is regarded as a simplified fuzzy reasoning model and called neurofuzzy.

KEYWORDS: ***neurofuzzy, optimal control, RBF, variational problem, direct solution method***

1. INTRODUCTION

J. Moody and C. J. Darken [1, 2] have proposed radial basis function (RBF) networks, a technique for interpolating in a high-dimensional space, and reported that the training of RBF networks is potentially 1000 times

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Received October 1994; accepted May 1995

International Journal of Approximate Reasoning 1995; 13:287–302

655 Avenue of the Americas, New York, NY 10010
0888-613X/95/$9.50
SSDI 0888-613X(95)00058-O

faster than for sigmoidal-basis-function networks with back propagation for comparable error rates. The RBF network can be regarded as a three-layered neural network [1–4] and a simplified fuzzy reasoning model [5–10]. In this paper we propose an optimal-control scheme for nonlinear systems that uses RBF networks, which we call a *neurofuzzy approach.* In the proposed method the cost function of an integral form is minimized by the steepest-descent method. The methodology shows promise for application in control problems that are so complex that analytical design techniques are not suitable. It is shown that the RBF networks can be used to solve highly nonlinear control problems.

In this paper we deal with a direct drive (DD) manipulator system and a trailer-truck system as severely nonlinear systems. The DD manipulator system is described by a continuous-time dynamical model, and the trailer-truck system is described by a discrete-time dynamical model.

For multijoint arm movement, there exist complicated control problems because of the presence of interactional forces such as coriolis forces and reaction forces. When the hand of the multijoint arm is moved from one position to another, there are infinitely many possible paths. Though the minimum-jerk model proposed by Flash and Hogan [11] takes into account the kinematics of movement, it is independent of the dynamics of the musculoskeltal system. Uno et al. [12] have proposed a performance index, viz. the sum of squares of the torque change rates integrated over the entire movement period. The model is called a *minimum-torque-change model.* Uno et al. have shown that the hand trajectories yielded by the minimum-torque-change model are in better agreement with human arm movement than is the minimum-jerk model. The iterative scheme for the minimum-torque-change model uses variational calculus and dynamic optimization theory. Hence, it seems to be a control-theoretic method rather than a neuroscientific one. In this paper, we propose a direct solution method of this variational problem using Gaussian RBFs, which can be reinterpreted as a simplified fuzzy reasoning model. The RBF networks are attractive in that they are potentially faster than the conventional back-propagation networks [13] for comparable error rates in supervised learning [1].

Control of a trailer truck backing to a loading dock [14, 15] is a difficult problem, for the system is nonlinear and unstable. The neural-network truck backer-upper control was developed by Nguyen and Widrow [16]. In their approach an emulator, a multilayered neural network [13], learns to identify the system's dynamics characteristics. A controller, another multilayered neural network, then is trained to minimize the final-state error and control energy. The advantage of this approach is in realizing an optimal feedback control based on a cost function of some state and manipulated variables. Unfortunately, however, a trained emulator is needed for this approach, and thousands of backups are required. Therefore, training the network using an actual trailer truck is not a realistic approach. Hence, the training is carried out by a computer simulation using a mathematical model of the trailer-truck dynamics.

We apply the proposed neurofuzzy scheme, which is a direct solution method, to the variational problem, and the trajectories of state and input variables of nonlinear systems are represented by Gaussian functions.

In Section 2 we describe a neuro-fuzzy optimal-control scheme, and in Section 3 we apply the proposed method to a first-order lag system which has an input variable and a state variable. Sections 4 and 5 are devoted to describing applications to a DD manipulator system and a trailer-truck system respectively.

2. NEUROFUZZY OPTIMAL CONTROL

Let $\mathbf{x} = (x_1, x_2, \ldots, x_Q)'$ be a vector of state variables in an optimal-control problem, where $'$ denotes transpose. Let $\mathbf{u} = (u_1, u_2, \ldots, u_R)'$ be an input vector of manipulated variables. Then the state equation is written as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u}). \tag{1}$$

Let $\mathbf{x}(0)$ be an initial state. T is an appropriately chosen time to terminate control. $\mathbf{x}(T)$ is a terminal state. The cost function is the following:

$$J = \int_0^T F(\mathbf{x}(t), \mathbf{u}(t))\, dt + G(\mathbf{x}(0), \mathbf{x}(T)). \tag{2}$$

We seek an optimal control by which the integral of F with respect to t from 0 to T and the initial- and final-state errors represented by G are minimized. Hence, it is a variational problem to find the function $\mathbf{x}(t)$ and $\mathbf{u}(t)$ which minimize the cost function J.

First, M independent variables are chosen from the state variables $(x_1, x_2, \ldots, x_Q)$ and the manipulated variables $(u_1, u_2, \ldots, u_R)$. Each independent variable is represented by Gaussian functions with one input variable t (time). The fuzzy reasoning if-then rules are written as

$$\text{if } t \text{ is } \mu_{mk} \text{ then } y_m \text{ is } w_{mk} \qquad (k = 1, \ldots, K),$$

where K is the number of fuzzy rules used for representing $y_m(t)$. The membership function of the premise part of each fuzzy rule for the independent variable $y_m(t)$ is defined by a Gaussian function (i.e. a bell-shaped membership function) as

$$\mu_{mk}(t) = \exp\left(- \frac{(t - a_{mk})^2}{b_{mk}} \right) \qquad (k = 1, \ldots, K). \tag{3}$$

The mth independent variable can be written as the fuzzy model

$$y_m(t) = \sum_{k=1}^{K} \mu_{mk}(t) w_{mk} \qquad (m = 1, 2, \ldots, M). \tag{4}$$

Here y_m is equivalent to Gaussian RBFs [1–4]. Let $\mathbf{y} = (y_1, y_2, \ldots, y_M)'$ be a vector of these independent variables. Then the other state and manipulated variables can be represented by the independent variables $y_1, y_2, \ldots, y_M$. Substituting $y_1, y_2, \ldots, y_M$ into the state and constraints equations, we have

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}), \tag{5}$$

$$\mathbf{g}(\mathbf{y}, \dot{\mathbf{y}}) = \mathbf{O}. \tag{6}$$

The cost function of the neurofuzzy optimal control can be written as

$$\begin{aligned} J = &\int_0^T F(\mathbf{y}(t), \dot{\mathbf{y}}(t))\, dt \\ &+ \alpha \int_0^T \| \dot{\mathbf{y}}(t) - \mathbf{f}(\mathbf{y}(t)) \|^2 \, dt \\ &+ \mathbf{G}(\mathbf{y}(0), \mathbf{y}(T), \dot{\mathbf{y}}(0), \dot{\mathbf{y}}(T)), \end{aligned} \tag{7}$$

where α is a positive constant. For numerical integration of the cost function, Sympson's formula is adopted. The learning rules based on the gradient descent method are

$$w_{mk}^{\mathrm{NEW}} = w_{mk}^{\mathrm{OLD}} - \tau \frac{\partial J}{\partial w_{mk}}, \tag{8}$$

$$a_{mk}^{\mathrm{NEW}} = a_{mk}^{\mathrm{OLD}} - \tau \frac{\partial J}{\partial a_{mk}}, \tag{9}$$

$$b_{mk}^{\mathrm{NEW}} = b_{mk}^{\mathrm{OLD}} - \tau \frac{\partial J}{\partial b_{mk}}, \tag{10}$$

where τ is the positive learning rate.

3. APPLICATION TO A FIRST-ORDER LAG SYSTEM

We apply the neurofuzzy optimal control to a simple first-order lag system and compare the result with the theoretical solution.

The state equation and the boundary conditions are given as

$$\dot{x}(t) = -cx(t) + u(t), \qquad x(0) = x_0, \quad x(T) = 0. \tag{11}$$

The problem is to find an optimal control by which the constraints in Equation (11) are satisfied and the cost function

$$\begin{aligned} J(x(t)) &= \int_0^T \{x^2(t) + u^2(t)\}\, dt \\ &= \int_0^T \{x^2(t) + [\dot{x}(t) + cx(t)]^2\}\, dt \end{aligned} \tag{12}$$

is minimized. Since the system is linear and the cost function is quadratic, (i.e. the LQ problem), we have a theoretical solution. By our proposed method,

$$x(t) = \sum_{k=1}^{K} \mu_k(t) w_k, \tag{13}$$

$$\mu_k(t) = \exp\left(-\frac{(t-a_k)^2}{b_k}\right), \tag{14}$$

$$\begin{aligned} J &= \int_0^T \{x^2(t) + [\dot{x}(t) + cx(t)]^2\}\, dt \\ &\quad + s_0[x(0) - x_0]^2 + s_T x^2(T), \end{aligned} \tag{15}$$

where s_0 and s_T are the positive constants for evaluating the errors in the initial condition and the terminal condition respectively. From Equation (13) we have

$$\dot{x}(t) = \sum_{k=1}^{K} \left(-\frac{2(t-a_k)}{b_k}\right) \mu_k(t) w_k. \tag{16}$$

The learning rules are as in Equations (8)–(10), and we have $c = 1$, $x_0 = 10$, $T = 4$, $K = 10$, $s_0 = 100$, $s_T = 100$. We use Simpson's formula of numerical integration. In Figure 1(a) the dashed line represents the computational result of an approximately optimal solution by the neurofuzzy approach. The solid line (theoretical solution) and the dashed line almost coincide.

The manipulated variable $u(t)$ *can* be obtained by the relation $u(t) = \dot{x}(t) + cx(t)$ and is shown in Figure 1(b). Both $x(t)$ and $u(t)$ (shown by the dashed lines) are similar to the theoretical solutions $x^*(t)$ and $u^*(t)$ (shown by the solid lines) respectively.

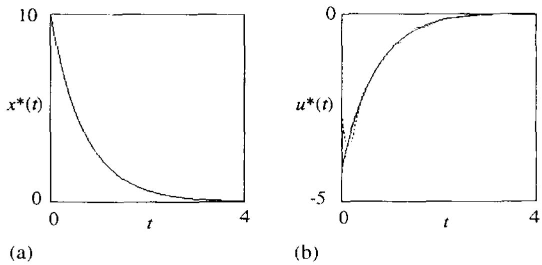

Figure 1. An approximately optimal solution by the neurofuzzy and the theoretical optimal solution ($c = 1$, $x_0 = 10$, $T = 4$).

4. OPTIMAL CONTROL OF A DD MANIPULATOR

4.1. Neurofuzzy Minimum-Torque-Change Model for a DD Manipulator

We consider a two-joint DD manipulator as shown in Figure 2, which moves within a horizontal plane. The manipulator dynamics is given as

$$\begin{aligned}\left[I_1 + I_2 + 2M_2L_1S_2\cos\theta_2 + M_2(L_1)^2 + J_1\right]\ddot{\theta}_1 \\ +(I_2 + M_2L_1S_2\cos\theta_2)\ddot{\theta}_2 \\ - M_2L_1S_2\left(2\dot{\theta}_1 + \dot{\theta}_2\right)\dot{\theta}_2\sin\theta_2 + r_1\dot{\theta}_1 = k_1v_1,\end{aligned} \tag{17}$$

$$\begin{aligned}(I_2 + M_2L_1S_2\cos\theta_2)\ddot{\theta}_1 + (I_2 + J_2)\ddot{\theta}_2 \\ + M_2L_1S_2\left(\dot{\theta}_2\right)^2\sin\theta_2 + r_2\dot{\theta}_2 = k_2v_2,\end{aligned} \tag{18}$$

where M_i, L_i, and S_i represent the mass, the length, and the distance from center of mass to joint, respectively, and I_i represents the rotary inertia of

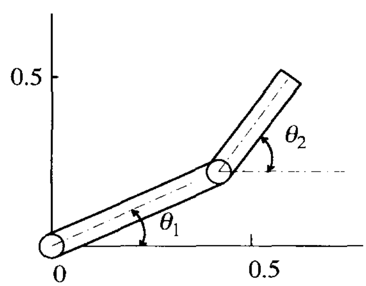

Figure 2. A two-joint manipulator which moves within a horizontal plane.

link i around the joint. r_i is the viscosity coefficient. The cost function is given as

$$J(\tau_1(t), \tau_2(t)) = \frac{1}{2}\left(\int_0^T \left[C_1(\dot{\tau}_1)^2 + C_2(\dot{\tau}_2)^2\right] dt\right.$$
$$+C_3\left[\theta_1^0 - \theta_1(0)\right]^2 + C_4\left[\theta_1^T - \theta_1(T)\right]^2$$
$$+C_5\left[\theta_2^0 - \theta_2(0)\right]^2 + C_6\left[\theta_2^T - \theta_2(t)\right]^2$$
$$+C_7[\dot{\theta}(0)]^2 + C_8\left[\dot{\theta}_2(0)\right]^2 + C_9\left[\dot{\theta}_1(T)\right]^2$$
$$+C_{10}\left[\dot{\theta}_2(T)\right]^2 + C_{11}\left[\ddot{\theta}_1(0)\right]^2 + C_{12}\left[\ddot{\theta}_2(0)\right]^2$$
$$\left.+C_{13}\left[\ddot{\theta}_1(T)\right]^2 + C_{14}\left[\ddot{\theta}_2(T)\right]^2\right), \tag{19}$$

where θ_i^0 represents the initial angle of the ith link and θ_i^T is the final angle of the ith link. T represents a given time for movement. The right-hand sides of the Equations (17) and (18) correspond to torques τ_i $(i = 1, 2)$ respectively. The derivatives of τ_i are

$$\dot{\tau}_1(t) = M_2 L_1 S_2\left\{-\left(2\dot{\theta}_1\ddot{\theta}_2 + 4\ddot{\theta}_1\dot{\theta}_2 + 3\dot{\theta}_2\ddot{\theta}_2\right)\sin\theta_2\right.$$
$$\left.+\left[2\dddot{\theta}_1 + \dddot{\theta}_2 - 2\dot{\theta}_1\left(\dot{\theta}_2\right)^2 - \left(\dot{\theta}_2\right)^3\right]\cos\theta_2\right\}$$
$$+\left[I_2 + I_2 + M_2(L_1)^2 + J_1\right]\dddot{\theta}_1 + I_2\dddot{\theta}_2 + r_1\ddot{\theta}_1 \tag{20}$$

and

$$\dot{\tau}_2(t) = M_2 L_1 S_2\left\{\left(2\dot{\theta}_1\ddot{\theta}_1 - \ddot{\theta}_1\dot{\theta}_2\right)\sin\theta_2\right.$$
$$\left.+\left[\left(\dot{\theta}_1\right)^2\dot{\theta}_2 + \dddot{\theta}_1\right]\cos\theta_2\right\}$$
$$+J_2\dddot{\theta}_2 + I_2\left(\dddot{\theta}_1 + \dddot{\theta}_2\right) + r_2\ddot{\theta}_2. \tag{21}$$

We define the joint angle $\theta_i(t)$ as

$$\theta_i(t) = \sum_{k=1}^{K} \mu_{ik}(t) w_{ik} \qquad (i = 1, 2), \tag{22}$$

$$\mu_{ik}(t) = \exp\left(-\frac{(t - a_{ik})^2}{b_{ik}}\right). \tag{23}$$

The initial values of the parameters in (16) are set as

$$a_{ik} = \frac{T}{K-3}(k-2), \tag{24}$$

$$b_{ik} = \frac{T}{2(K-3)}, \tag{25}$$

$$w_{ik} = 0.0, \tag{26}$$

where T represents a given time for movement. K is the number of fuzzy rules (Gaussian functions).

4.2. Trajectory Formation by the Gradient Descent Method

The physical parameters of the manipulator are given in Table 1. The movement from $\theta_1 = \theta_2 = 0.0$ rad to $\theta_1 = \theta_2 = 1.0$ rad with the duration of one second is assumed. The weight parameters C_i ($i = 1$–14) of the cost function are given in Table 2.

The computational results for the 1st link are shown in Figure 3. The trajectories are depicted by solid lines, and the numbers of learning iterations are also shown in the figures. Figure 4 shows the change of the value of the cost function as learning proceeds. Table 3 shows errors at the initial and terminal positions. Figure 5 shows the trajectory passing through a via point.

5. TRAILER-TRUCK BACKER-UPPER

Figure 6 shows a diagram of a trailer and truck system. The definition of the state variables (ϕ, ψ, θ, η, and ζ) and the manipulated variable (δ) is also illustrated in Figure 6. The problem is to control the steering of a

Table 1. Values of the Physical Parameters of the Manipulator Shown in Figure 2

Parameter	Link 1	Link 2
M_i (kg)	3.0	2.0
L_i (m)	0.50	0.35
S_i (m)	0.21	0.15
I_i ($\text{kg} \cdot \text{m}^2$)	0.27	0.10
J_i ($\text{kg} \cdot \text{m}^2$)	0.0005	0.0003
r_i ($\text{kg} \cdot \text{m}^2/\text{s}$)	0.20	0.15
k_i ($\text{N} \cdot \text{m/V}$)	0.30	0.10

Table 2. Values of the Coefficients of the Cost Function

Learning rate η	0.000001		
C_1	0.01	C_2	0.01
C_3	50,000	C_4	50,000
C_5	50,000	C_6	50,000
C_7	10,000	C_8	10,000
C_9	10,000	C_{10}	10,000
C_{11}	100	C_{12}	100
C_{13}	100	C_{14}	100

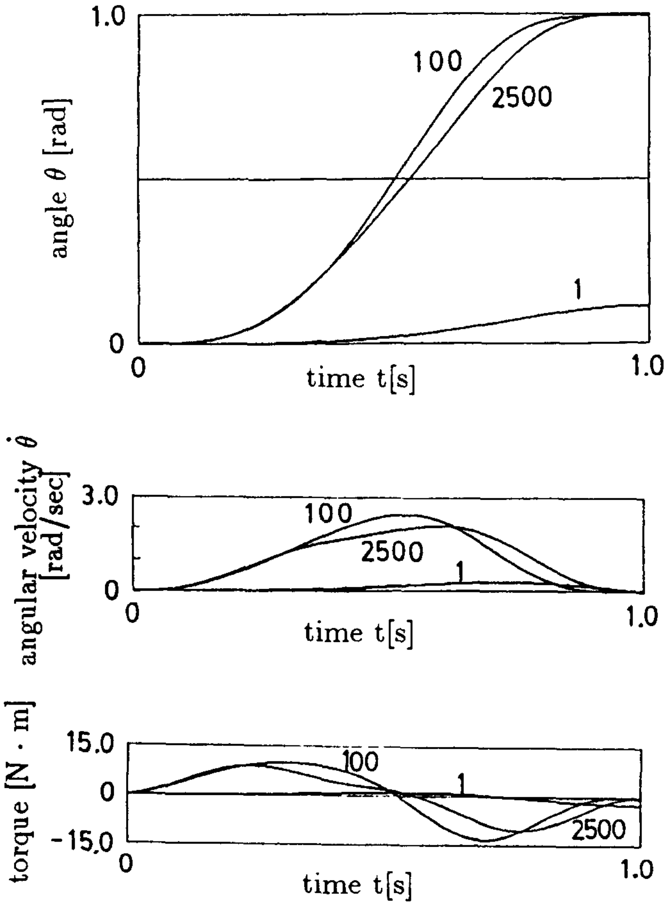

Figure 3. Trajectories of the first link.

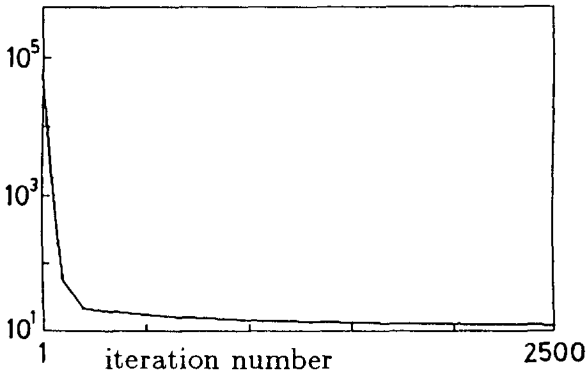

Figure 4. Changes in the cost-function value as learning proceeds.

trailer truck while backing up to a loading dock from an initial position. Only backing up is allowed. It is assumed that the truck moves very slowly. Let time step Δt be small; then the dynamical equations of the trailer-truck system can be written geometrically as

$$\psi[i + 1] = \psi[i] + \frac{v\,\Delta t \tan \delta[i]}{l}, \tag{27}$$

$$\theta[i + 1] = \theta[i] + \frac{v\,\Delta t \sin \phi[i]}{L}, \tag{28}$$

$$\zeta[i + 1] = \zeta[i] + v\,\Delta t \cos \phi[i] \cos \frac{\theta[i + 1] + \theta[i]}{2}, \tag{29}$$

$$\eta[i + 1] = \eta[i] + v\,\Delta t \cos \phi[i] \sin \frac{\theta[i + 1] + \theta[i]}{2}, \tag{30}$$

$$\phi[i] = \psi[i] - \theta[i]. \tag{31}$$

Table 3. Errors at the Initial and Terminal Positions (Absolute Values)

$\theta_1(0.0)$	0.000370	$\theta_2(0.0)$	0.000112
$\dot{\theta}_1(0.0)$	0.000430	$\dot{\theta}_2(0.0)$	0.000107
$\ddot{\theta}_1(0.0)$	0.015510	$\ddot{\theta}_2(0.0)$	0.003999
$\theta_1(1.0)$	0.000430	$\theta_2(1.0)$	0.000036
$\dot{\theta}_1(1.0)$	0.000622	$\dot{\theta}_2(1.0)$	0.000112
$\ddot{\theta}_1(1.0)$	0.022390	$\ddot{\theta}_2(1.0)$	0.000519

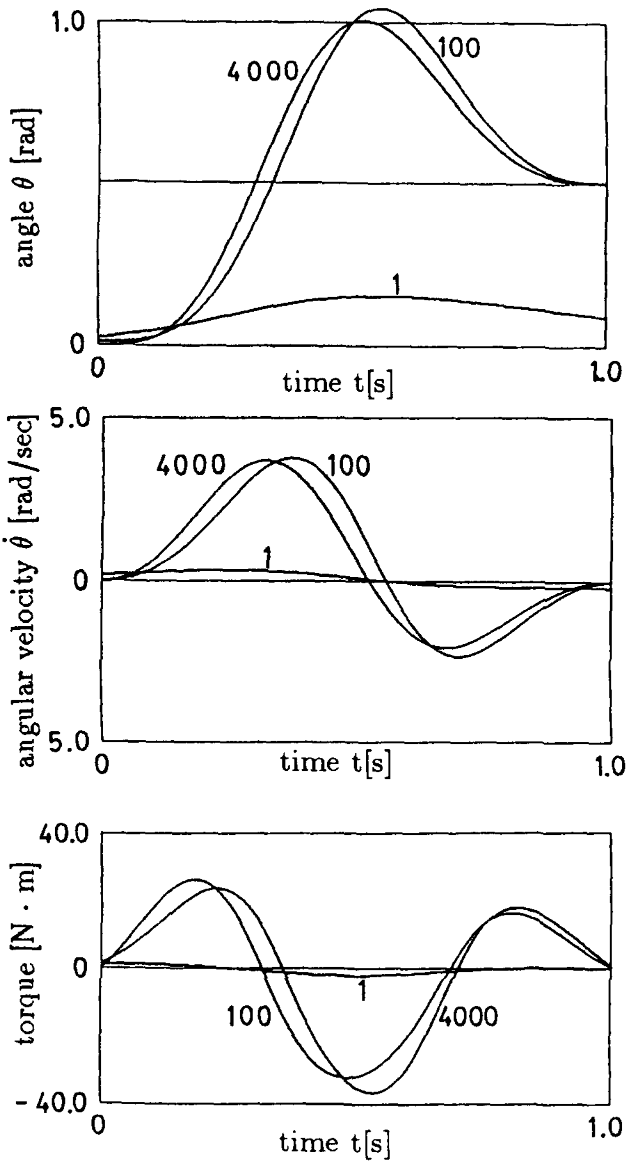

Figure 5. Obtained trajectories of the first link passing through a via point.

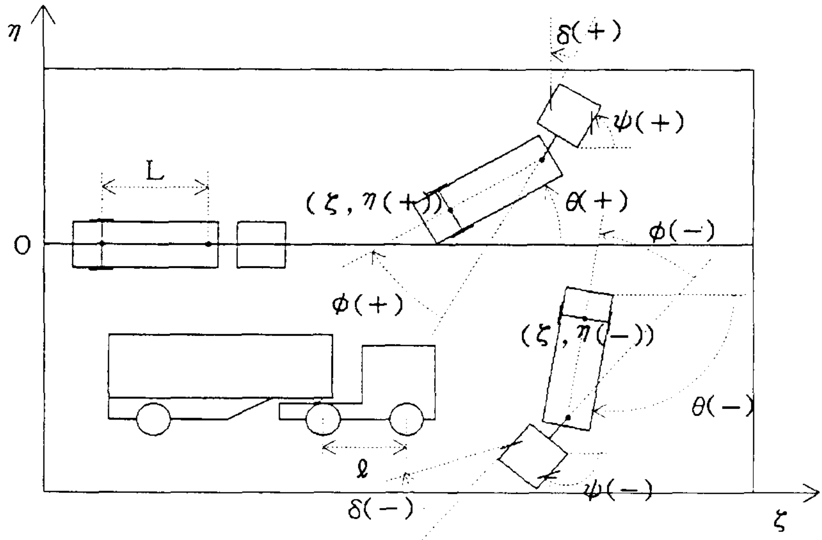

Figure 6. Diagram of a truck and trailer. ψ = angle of the truck with horizontal, θ = angle of the trailer with horizontal, ϕ = relative angle of the truck and trailer, δ = steering angle, (η, ζ) = Cartesian coordinates of the robot.

From above difference equations (27)–(31), the four variables ψ, θ, η, and δ can be written in terms of ϕ alone as

$$\phi[i] = \theta_0 + \sum_{n=0}^{i-1} \frac{v\,\Delta t \sin \phi[i-1-n]}{L}, \tag{32}$$

$$\eta[i] = \eta_0 + \sum_{n=0}^{i-1} \left(v\,\Delta t \cos \phi[i-1-n] \sin \frac{\theta[i-n] + \theta[i-1-n]}{2} \right), \tag{33}$$

$$\delta[i] = \tan^{-1}\left(\frac{l \cdot (\phi[i+1] - \phi[i] + \theta[i+1] - \theta[i])}{v\,\Delta t} \right), \tag{34}$$

where θ_0 and η_0 are the initial values of θ and η respectively. $\theta[i]$ in Equations (33) and (34) can be replaced with Equation (32). Hence, only the relative angle ϕ of the truck and the trailer is represented as a neurofuzzy model:

$$\phi[i] = \sum_{k=1}^{K} \mu_k[i] \cdot w_k, \tag{35}$$

$$\mu_k[i] = \exp\left(-\frac{(i\,\Delta t - a_k)^2}{b_k} \right). \tag{36}$$

The goal is to make the back of the trailer parallel to the loading dock and to have θ, ψ, and η equal zero with as little steering as possible. By substituting Equations (35) and (36) into Equation (32)–(34), we have a cost function of quadratic form:

$$J = \sum_{i=0}^{N-1} (q_1\phi^2[i] + q_2\theta^2[i] + q_2\eta^2[i] + r\delta^2[i])$$

$$+ s_0(\phi[0] - \phi_0)^2 + s_1\phi^2[N] + s_2\theta^2[N] + s_2\eta^2[N], \tag{37}$$

where ϕ_0 is the given initial value of ϕ. It should be noted that the unknown parameters in Equation (37) are w_k, a_k, and b_k ($k = 1, \ldots, K$). The coefficients q_1, q_2, q_3, and r are the positive weights for $\phi[i]$, $\theta[i]$, $\eta[i]$, and $\delta[i]$ respectively. s_0 and s_i ($i = 1, 2, 3$) are the positive weights for the initial condition and the terminal condition respectively. The learning rules are Equations (8)–(10), and the parameters are given in Table 4. We set $t_1 = 0$ and $t_2 = 1$. Figure 7 shows the computational result. Figure 8 shows the simulation result of the followup control. Figure 9 shows another simulation result where the weights in the cost function were changed to $q_1 = r = 0.001$, $q_2 = 0.01$, $q_3 = 0.0001$, $s_0 = s_1 = s_2 = 1000.0$, and $s_3 = 1.0$.

Table 4. The Parameter Values

τ	0.000001	K	15
q_1	0.01	l	2.8 m
q_2	0.1	L	5.5 m
q_3	0.0001	v	-1.0 m/s
r	0.01	Δt	2.0 s
s_0	1000.0	N	50
s_1	100.0	ϕ_0	0.0°
s_2	100.0	θ_0	$-135.0°$
s_3	0.01	η_0	10.0 m

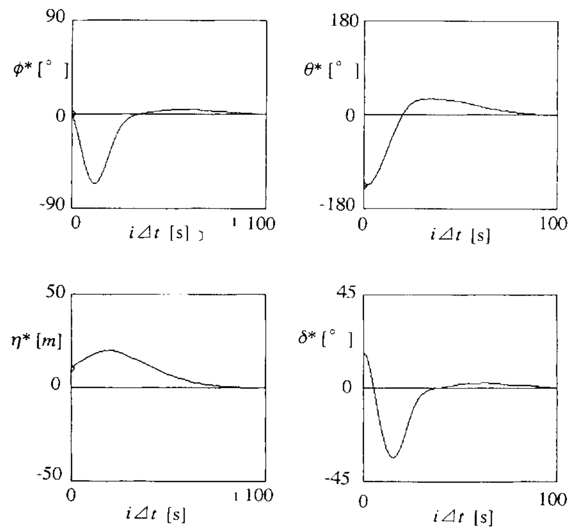

Figure 7. An approximately optimal control of backing up a trailer truck by the neurofuzzy approach.

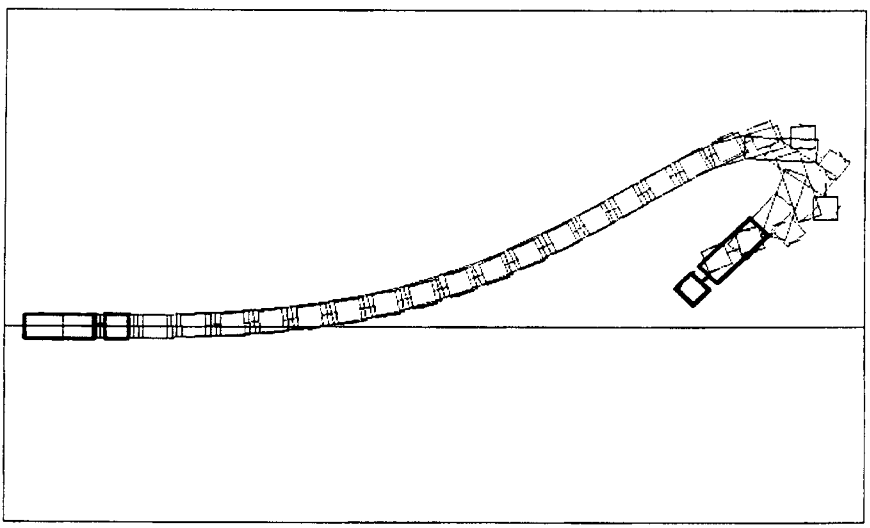

Figure 8. A locus of the trailer truck following up to the optimal trajectory.

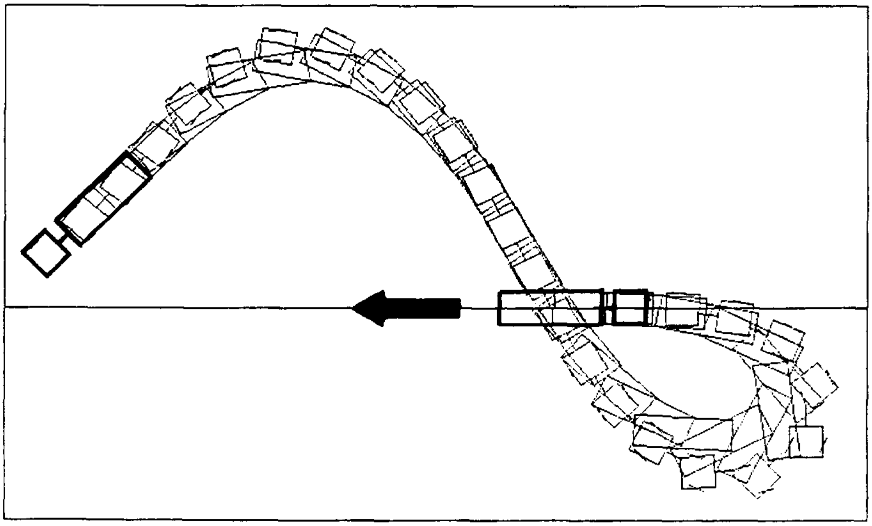

Figure 9. Simulation result with $q_1 = r = 0.001$, $q_2 = 0.01$, $q_3 = 0.0001$, $s_0 = s_1 = s_2 = 1000.0$, and $s_3 = 1.0$.

6. CONCLUDING REMARKS

We have proposed a direct solution method for variational problems. The trajectories of state and manipulated variables are represented by networks of Gaussian functions, which can be reinterpreted as simplified fuzzy reasoning rules. In conventional fuzzy control, the parameter tuning of fuzzy rules is a troublesome problem, since it is a time consuming task for engineers. The proposed method, which is based on the mathematical models of a control object, may present a convenient form for this optimizing procedure and provides an easy-to-use technique for engineers.

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